Дополнителни вежби по Математика 2, 09.03.2023

Задача 1, Тест 1 (Т1) и Тест 2 (Т2) од 17.06.2022.

1.a.(T1)

$$\int\left(30x^5+\frac{4}{x^2+16}-3^{x-2}+1\right)dx=\int 30x^5dx+\int\frac{4}{\underbrace{x^2+16}_{=4^2}}dx-\int \underbrace{3^{x-2}}_{=3^x\cdot 3^{-2}}dx+\int dx$$

$$=30\int x^5dx+4\int\frac{1}{x^2+4^2}dx-3^{-2}\int 3^x dx+\int dx$$

$$=30\frac{x^{5+1}}{5+1}+4\cdot\frac{1}{4}\operatorname{arctg}\frac{x}{4}-3^{-2}\frac{3^x}{\ln 3}+x+C$$

$$=5x^6+\operatorname{arctg}\frac{x}{4}-\frac{3^{x-2}}{\ln 3}+x+C.$$

Напомена: Наместо $\frac{4}{x^2+16}$ може да стои:

1. $\frac{4x}{x^2\pm 16} \Rightarrow \int\frac{4x}{x^2\pm 16}dx=4\int\frac{x}{x^2\pm 16}dx=4\cdot\frac{1}{2}\ln\left|x^2\pm 16\right|=2\ln\left|x^2\pm 16\right|$, или
2. $\frac{4}{x^2-16} \Rightarrow \int\frac{4}{x^2-16}dx=4\int\frac{dx}{x^2-16}=\cancel{4}\cdot\frac{1}{2\cdot\cancel{4}}\ln\left|\frac{x-\sqrt{16}}{x+\sqrt{16}}\right|=\frac{1}{2}\ln\left|\frac{x-4}{x+4}\right|$.

1.a.(T2)

$$\int\left(12x^5-\frac{5}{\sqrt{25-x^2}}+2^{x+1}+1\right)dx=\int 12x^3dx-\int\frac{5}{\sqrt{\underbrace{25}_{=5^2}-x^2}}dx+\int \underbrace{2^{x+1}}_{=2^x\cdot 2}dx+\int dx$$

$$=12\int x^3dx-5\int\frac{1}{\sqrt{5^2-x^2}}dx+2\int 2^x dx+\int dx$$

$$=12\frac{x^{3+1}}{3+1}-5\arcsin\frac{x}{5}+2\frac{2^x}{\ln 2}+x+C$$

$$=3x^4-5\arcsin\frac{x}{5}+\frac{2^{x+1}}{\ln 2}+x+C.$$

1.б.(T1)

$$\int(9x-1)\ln x dx=\left|\begin{array}{l}\text{парц. инт.}\\ u=\ln x,\quad dv=(9x-1)dx\\ du=\frac{dx}{x},\quad v=\int(9x-1)dx=9\cdot\frac{x^2}{2}-x\end{array}\right|$$

$$=\left(9\cdot\frac{x^2}{2}-x\right)\ln x-\int\left(9\cdot\frac{x^2}{2}-x\right)\frac{dx}{x}=\left(9\cdot\frac{x^2}{2}-x\right)\ln x-\int\left(9\cdot\frac{x}{2}-1\right)\frac{dx}{x}$$

$$=\left(9\cdot\frac{x^2}{2}-x\right)\ln x-\left(\frac{9}{2}\cdot\frac{x^2}{2}-x\right)+C=x\left(\frac{9}{2}x-1\right)\ln x-\left(\frac{9}{4}x-1\right)x+C$$

1.б.(T2)

$$\int (8x+1)\sin x dx = \left| \begin{array}{l} \text{парц. инт.} \\ u = 8x+1, \quad dv = \sin dx \\ du = 8dx, \quad v = \int \sin x dx = -\cos x \end{array} \right|$$

$$= -(8x+1)\cos x - \int (-\cos x)\cdot 8dx = -(8x+1)\cos x + 8\int \cos x dx$$

$$= -(8x+1)\cos x + 8\sin x + C$$

1.в.(T1)

$$\int \frac{x-2}{\sqrt{x^2+4x+10}}dx = \int \frac{x-2}{\sqrt{x^2+4x+4+6}}dx = \int \frac{x-2}{\sqrt{(x+2)^2+6}}dx = \left| \begin{array}{l} \text{смена: } x+2=t \\ dx = dt, \\ x = t-2 \end{array} \right|$$

$$= \int \frac{t-2-2}{\sqrt{t^2+6}}dt = \int \frac{t}{\sqrt{t^2+6}}dt - 4\int \frac{dt}{\sqrt{t^2+6}}$$

$$= \sqrt{t^2+6} - 4\ln\left|t+\sqrt{t^2+6}\right| + C$$

$$= \sqrt{x^2+4x+10} - 4\ln\left|x+2+\sqrt{x^2+4x+10}\right| + C$$

1.в.(T2)

$$\int \frac{x+3}{\sqrt{x^2-6x+4}}dx = \int \frac{x+3}{\sqrt{x^2-6x+9-9+4}}dx = \int \frac{x+3}{\sqrt{(x-3)^2-5}}dx = \left| \begin{array}{l} \text{смена: } x-3=t \\ dx = dt, \\ x = t+3 \end{array} \right|$$

$$= \int \frac{t+3+3}{\sqrt{t^2-5}}dt = \int \frac{t}{\sqrt{t^2-5}}dt + 6\int \frac{dt}{\sqrt{t^2-5}}$$

$$= \sqrt{t^2-5} + 6\ln\left|t+\sqrt{t^2-5}\right| + C$$

$$= \sqrt{x^2-6x+4} + 6\ln\left|x-3+\sqrt{x^2-6x+4}\right| + C$$

1.г.(T1)

$$\int \frac{dx}{7+5\sin^2 x} = \int \frac{dx}{7(\sin^2 x+\cos^2 x)+5\sin^2 x} = \int \frac{dx}{12\sin^2 x + 7\cos^2 x}$$

$$= \int \frac{dx}{\left[12\left(\frac{\sin x}{\cos x}\right)^2 + 7\right]\cos^2 x} = \left| \begin{array}{l} \text{смена: } \operatorname{tg} x = t \\ \frac{dx}{\cos^2 x} = dt \end{array} \right| = \int \frac{dt}{12t^2+7}$$

$$= \frac{1}{12}\int \frac{dt}{t^2+\left(\frac{\sqrt{7}}{\sqrt{12}}\right)^2} = \frac{1}{12}\cdot\frac{1}{\frac{\sqrt{7}}{\sqrt{12}}}\operatorname{arctg}\frac{t}{\frac{\sqrt{7}}{\sqrt{12}}} + C = \frac{1}{\sqrt{12}\cdot\sqrt{7}}\operatorname{arctg}\frac{\sqrt{12}}{\sqrt{7}}t + C$$

$$= \frac{1}{2\sqrt{21}}\operatorname{arctg}\frac{2\sqrt{3}}{\sqrt{7}}\operatorname{tg} x + C$$

1.г.(T2)

$$\int \frac{dx}{9+4\cos^2 x} = \int \frac{dx}{9(\sin^2 x+\cos^2 x)+4\cos^2 x} = \int \frac{dx}{9\sin^2 x+13\cos^2 x}$$

$$= \int \frac{dx}{\left[9\left(\frac{\sin x}{\cos x}\right)^2+13\right]\cos^2 x} = \left| \begin{array}{l} \text{смена: } \operatorname{tg} x = t \\ \frac{dx}{\cos^2 x} = dt \end{array} \right| = \int \frac{dt}{9t^2+13}$$

$$= \frac{1}{9}\int \frac{dt}{t^2+\left(\frac{\sqrt{13}}{3}\right)^2} = \frac{1}{9}\cdot\frac{1}{\frac{\sqrt{13}}{3}}\operatorname{arctg}\frac{t}{\frac{\sqrt{13}}{3}}+C = \frac{1}{3\sqrt{13}}\operatorname{arctg}\frac{3t}{\sqrt{13}}+C$$

$$= \frac{1}{3\sqrt{13}}\operatorname{arctg}\frac{3\operatorname{tg} x}{\sqrt{13}}+C$$