① а) $A \cdot B + \frac{9}{5} C = ?$ ако

$$A = \begin{bmatrix} -1 & -3 & 0 \\ 0 & 4 & -2 \end{bmatrix}; \quad B = \begin{bmatrix} -7 & 4 \\ 2 & 0 \\ 0 & -5 \end{bmatrix}; \quad C = \begin{bmatrix} 5 & -10 \\ 15 & 0 \end{bmatrix}$$

б) $\frac{5}{6} A + B \cdot C = ?$ ако

$$A = \begin{bmatrix} 36 & 0 \\ -6 & 18 \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 2 & 0 \\ -3 & 0 & 1 \end{bmatrix}; \quad C = \begin{bmatrix} -8 & 0 \\ 2 & 1 \\ 0 & -1 \end{bmatrix}$$

Решение:

а) $$A \cdot B = \underset{2\times 3}{\begin{bmatrix} -1 & -3 & 0 \\ 0 & 4 & -2 \end{bmatrix}} \cdot \underset{3\times 2}{\begin{bmatrix} -7 & 4 \\ 2 & 0 \\ 0 & -5 \end{bmatrix}} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \overset{d_{11}}{(-1)\cdot(-7) + (-3)\cdot 2 + 0\cdot 0} & \overset{d_{12}}{(-1)\cdot 4 + (-3)\cdot 0 + 0\cdot(-5)} \\ \underset{d_{21}}{0\cdot(-7) + 4\cdot 2 + (-2)\cdot 0} & \underset{d_{22}}{0\cdot 4 + 4\cdot 0 + (-2)\cdot(-5)} \end{bmatrix}$$

$$= \begin{bmatrix} 7-6+0 & -4+0+0 \\ 0+8+0 & 0+0+10 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 8 & 10 \end{bmatrix} = A \cdot B$$

$$\frac{9}{5} C = \frac{9}{5} \cdot \begin{bmatrix} 5 & -10 \\ 15 & 0 \end{bmatrix} = \begin{bmatrix} \frac{9}{\cancel{5}} \cdot \cancel{5} & \frac{9}{5} \cdot (-10) \\ \frac{9}{\cancel{5}} \cdot \cancel{15}^{3} & \frac{9}{5} \cdot 0 \end{bmatrix} = \begin{bmatrix} 9 & -18 \\ 27 & 0 \end{bmatrix}$$

$$AB + \frac{9}{5} C = \begin{bmatrix} 1 & -4 \\ 8 & 10 \end{bmatrix} + \begin{bmatrix} 9 & -18 \\ 27 & 0 \end{bmatrix} = \begin{matrix} 1+9 & (-4)+(-18) \\ 8+27 & 10+0 \end{matrix}$$

$$= \begin{bmatrix} 10 & -22 \\ 35 & 10 \end{bmatrix}$$

б) $\frac{5}{6}A = \frac{5}{6}\cdot\begin{bmatrix} 36 & 0 \\ -6 & 18 \end{bmatrix} = \begin{bmatrix} \frac{5}{6}\cdot 36 & \frac{5}{6}\cdot 0 \\ \frac{5}{6}\cdot(-6) & \frac{5}{6}\cdot 18 \end{bmatrix} = \begin{bmatrix} 30 & 0 \\ -5 & 15 \end{bmatrix}$

$$B\cdot C = \begin{bmatrix} 0 & 2 & 0 \\ -3 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} -8 & 0 \\ 2 & 1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\cdot(-8)+2\cdot 2+0\cdot 0 & 0\cdot 0+2\cdot 1+0\cdot(-1) \\ (-3)\cdot(-8)+0\cdot 2+1\cdot 0 & (-3)\cdot 0+0\cdot 1+1\cdot(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 0+4+0 & 0+2+0 \\ 24+0+0 & 0+0+(-1) \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 24 & -1 \end{bmatrix}$$

$$\frac{5}{6}A + B\cdot C = \begin{bmatrix} 30 & 0 \\ -5 & 15 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 24 & -1 \end{bmatrix} = \begin{bmatrix} 34 & 2 \\ 19 & 14 \end{bmatrix}$$

2. Со метод на детерминанти (Крамерови формули) да се реши системот

$$\begin{cases} 7x + 2y + 4z = -6 \\ 9y + 2z = x + 6 \\ x + y = -2 \end{cases}$$

Решение:

$$\Leftrightarrow \begin{cases} 7x + 2y + 4z = -6 \\ -x + 9y + 2z = 6 \\ x + y + 0\cdot z = -2 \end{cases}$$

$$\begin{array}{ccccccc} & x & y & z & & & \\ D = & \left| \begin{matrix} 7 & 2 & 4 \\ -1 & 9 & 2 \\ 1 & 1 & 0 \end{matrix} \right. & & & \left| \begin{matrix} 7 & 2 \\ -1 & 9 \\ 1 & 1 \end{matrix} \right. \end{array}$$

$$= 7\cdot 9\cdot 0 + 2\cdot 2\cdot 1 + 4\cdot(-1)\cdot 1 - 1\cdot 9\cdot 4 - 1\cdot 2\cdot 7 - 0\cdot(-1)\cdot 2$$

$$= 0 + 4 - 4 - 36 - 14 - 0 = -50 \neq 0$$

$$D_x = \left|\begin{array}{ccc|cc} -6 & 2 & 4 & -6 & 2 \\ 6 & 9 & 2 & 6 & 9 \\ -2 & 1 & 0 & -2 & 1 \end{array}\right.$$

$$= (-6)\cdot 9\cdot 0 + 2\cdot 2\cdot(-2) + 4\cdot 6\cdot 1 - (-2)\cdot 9\cdot 4 - 1\cdot 2\cdot(-6) - 0\cdot 6\cdot 2$$

$$= 0 - 8 + 24 + 72 + 12 - 0 = 100$$

$$\boxed{x = \frac{D_x}{D} = \frac{100}{-50} = -2}$$

$$D_y = \left|\begin{array}{ccc|cc} 7 & -6 & 4 & 7 & -6 \\ -1 & 6 & 2 & -1 & 6 \\ 1 & -2 & 0 & 1 & -2 \end{array}\right.$$

$$= 7\cdot 6\cdot 0 + (-6)\cdot 2\cdot 1 + 4\cdot(-1)\cdot(-2) - 1\cdot 6\cdot 4 - (-2)\cdot 2\cdot 7 - 0\cdot(-1)\cdot(-6)$$

$$= 0 - 12 + 8 - 24 + 28 - 0 = 0$$

$$\boxed{y = \frac{D_y}{D} = \frac{0}{-50} = 0}$$

$$D_z = \left|\begin{array}{ccc|cc} 7 & 2 & -6 & 7 & 2 \\ -1 & 9 & 6 & -1 & 9 \\ 1 & 1 & -2 & 1 & 1 \end{array}\right.$$

$$= 7\cdot 9\cdot(-2) + 2\cdot 6\cdot 1 + (-6)\cdot(-1)\cdot 1 - 1\cdot 9\cdot(-6) - 1\cdot 6\cdot 7 - (-2)\cdot(-1)\cdot 2$$

$$= -126 + 12 + 6 + 54 - 42 - 4 = -100$$

$$\boxed{z = \frac{D_z}{D} = \frac{-100}{-50} = 2}$$

③ Да се најде кофакторната матрица $\tilde{A}$ и со нејзина помош да се одреди инверзната матрица A^{-1} ако

$$A = \begin{bmatrix} 1 & 4 & -2 \\ 0 & -1 & 3 \\ 2 & 8 & -3 \end{bmatrix}$$

Решение:

$$\tilde{A} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$A_{ij} = (-1)^{i+j} M_{ij}$, M_{ij} – минор што кореспондира на a_{ij}

$i,j \in \{1,2,3\}$

ако $\det(A) \neq 0$ тогаш постои A^{-1}

$$A^{-1} = \frac{1}{\det(A)} \tilde{A}^{T}$$

$$\det(A) = \begin{vmatrix} 1 & 4 & -2 \\ 0 & -1 & 3 \\ 2 & 8 & -3 \end{vmatrix} \begin{matrix} 1 & 4 \\ 0 & -1 \\ 2 & 8 \end{matrix}$$

$$= 1\cdot(-1)\cdot(-3) + 4\cdot 3\cdot 2 + (-2)\cdot 0\cdot 8 - 2\cdot(-1)\cdot(-2) - 8\cdot 3\cdot 1 - (-3)\cdot 0\cdot 4$$

$$= 3 + \cancel{24} + 0 - 4 - \cancel{24} - 0 = -1 \neq 0 \Rightarrow \exists!\, A^{-1}$$

$$M_{11} = \begin{vmatrix} -1 & 3 \\ 8 & -3 \end{vmatrix} = (-1)\cdot(-3) - 8\cdot 3 = 3 - 24 = -21 \Rightarrow A_{11} = \underbrace{(-1)^{1+1}}_{=1} M_{11} = M_{11} = -21$$

$$M_{12} = \begin{vmatrix} 0 & 3 \\ 2 & -3 \end{vmatrix} = 0\cdot(-3) - 2\cdot 3 = 0 - 6 = -6 \Rightarrow A_{12} = \underbrace{(-1)^{1+2}}_{-1} \cdot M_{12} = -M_{12} = 6$$

$$M_{13} = \begin{vmatrix} 0 & -1 \\ 2 & 8 \end{vmatrix} = 0 - (-2) = 2 \Rightarrow A_{13} = (-1)^{1+3} \cdot M_{13} = M_{13} = 2$$

$$M_{21} = \begin{vmatrix} 4 & -2 \\ 8 & -3 \end{vmatrix} = -12 - (-16) = 4 \Rightarrow A_{21} = (-1)^{2+1} M_{21} = -M_{21} = -4$$

$$M_{22} = \begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix} = -3-(-4) = 1 \Rightarrow A_{22} = (-1)^{2+2} M_{22} = M_{22} = 1$$

$$M_{23} = \begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix} = 8-8 = 0 \Rightarrow A_{23} = (-1)^{2+3} M_{23} = -M_{23} = 0$$

$$M_{31} = \begin{vmatrix} 4 & -2 \\ -1 & 3 \end{vmatrix} = 12-2 = 10 \Rightarrow A_{31} = (-1)^{3+1} M_{31} = M_{31} = 10$$

$$M_{32} = \begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} = 3-0 = 3 \Rightarrow A_{32} = (-1)^{3+2} M_{32} = -M_{32} = -3$$

$$M_{33} = \begin{vmatrix} 1 & 4 \\ 0 & -1 \end{vmatrix} = -1-0 = -1 \Rightarrow A_{33} = (-1)^{3+3} M_{33} = M_{33} = -1$$

$$\tilde{A} = \begin{bmatrix} -21 & 6 & 2 \\ -4 & 1 & 0 \\ 10 & -3 & -1 \end{bmatrix}$$ – кофакторна матрица на A

$$A^{-1} = \frac{1}{\det(A)} \tilde{A}^T = \frac{1}{-1} \cdot \begin{bmatrix} -21 & 6 & 2 \\ -4 & 1 & 0 \\ 10 & -3 & -1 \end{bmatrix}^T$$

$$= (-1) \cdot \begin{bmatrix} -21 & -4 & 10 \\ 6 & 1 & -3 \\ 2 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 21 & 4 & -10 \\ -6 & -1 & 3 \\ -2 & 0 & 1 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 4 & -2 \\ 0 & -1 & 3 \\ 2 & 8 & -3 \end{bmatrix} \cdot \begin{bmatrix} 21 & 4 & -10 \\ -6 & -1 & 3 \\ -2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21-24+4 & 4-4+0 & -10+12-2 \\ 0+6-6 & 0+1+0 & 0-3+3 \\ 42-48+6 & 8-8+0 & -20+24-3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

$A^{-1}A \overset{?}{=} I_3$ (проверка за дома)