① Со помош на Хорнерова шема да се определи количникот и остатокот од делењето на полиномот

а) $P(x)=x^5-4x^4+7x^2+9$ со биномот $x-3$ (за дома $x+3$)

б) $P(x)=x^5-5x^3+4x^2+6$ со биномот $x+2$ (за дома $x-2$)

Решение:

а) $P(x)=x^5-4x^4+0\cdot x^3+7x^2+0\cdot x+9$ $\qquad x-3=x-x_0 \Rightarrow x_0=3$

| | | | | | | |
|---|---|---|---|---|---|---|
| 3 | 1 | −4 | 0 | 7 | 0 | 9 |
| + | ↓ | 3 | −3 | −9 | −6 | −18 |
| | 1 | −1 | −3 | −2 | −6 | $\boxed{-9=P(3)\text{ – остаток}}$ |
| | $x^4$ | $x^3$ | $x^2$ | $x$ | 1 | |

$$P(x)=(x-3)\underbrace{(x^4-x^3-3x^2-2x-6)}_{\text{количник}}+\underbrace{(-9)}_{\text{остаток}}$$

б) $P(x)=x^5+0\cdot x^4-5x^3+4x^2+0\cdot x+6$ $\qquad x+2=x-x_0 \Rightarrow x_0=-2$

| | | | | | | |
|---|---|---|---|---|---|---|
| −2 | 1 | 0 | −5 | 4 | 0 | 6 |
| + | ↓ | −2 | 4 | 2 | −12 | 24 |
| | 1 | −2 | −1 | 6 | −12 | $\boxed{30=P(-2)\text{ – остаток}}$ |

$$P(x)=(x+2)\underbrace{(x^4-2x^3-x^2+6-12)}_{\text{количник}}+\underbrace{30}_{\text{остаток}}$$

② Да се определат дефиниционите области на следните функции

а) $f(x)=\dfrac{1}{x^2-4x}+\sqrt{\dfrac{x-2}{4x+7}}$ $\qquad$ б) $f(x)=\dfrac{1}{x^2+3x}+\ln\left(\dfrac{3x+1}{2x-5}\right)$

а) $$f(x) = \underbrace{\frac{1}{x^2-4x}}_{f_1(x)} + \underbrace{\sqrt{\frac{x-2}{4x+7}}}_{f_2(x)}$$

$$D_f = D_{f_1} \cap D_{f_2}$$

$D_{f_1}$: $\quad x^2 - 4x \neq 0$

$$x^2 - 4x = 0 \Rightarrow x(x-4) = 0$$
$$\Rightarrow x_1 = 0, \quad x_2 = 4$$

$$\Rightarrow D_{f_1} = \mathbb{R} \setminus \{0; 4\} = (-\infty; 0) \cup (0; 4) \cup (4; +\infty)$$

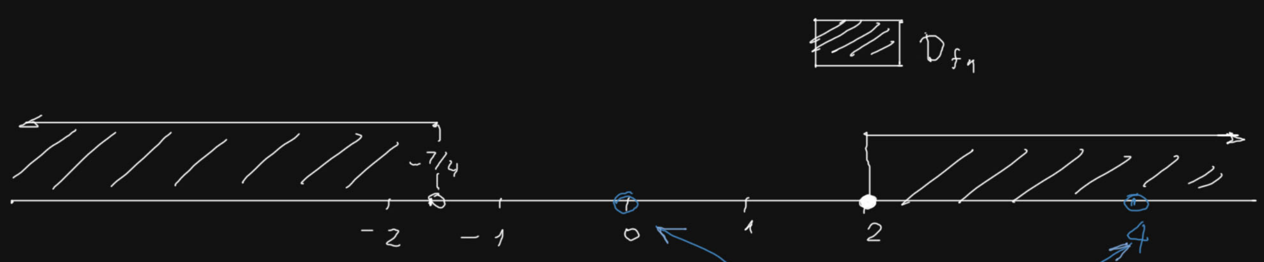
$D_{f_2}$: $\quad \dfrac{x-2}{4x+7} \geq 0, \quad 4x+7 \neq 0 \quad \left(\Rightarrow x \neq -\frac{7}{4}\right)$

$$D_{f_2} = A \cup B$$

$$A: \begin{cases} x-2 \geq 0 \\ 4x+7 > 0 \end{cases} \Rightarrow \begin{cases} x \geq 2 \\ 4x > -7 \quad / :4 \end{cases} \Rightarrow \begin{cases} x \geq 2 \\ x > -\frac{7}{4} \end{cases} \Rightarrow A = [2; +\infty)$$

$$B: \begin{cases} x-2 \leq 0 \\ 4x+7 < 0 \end{cases} \Rightarrow \begin{cases} x \leq 2 \\ x < -\frac{7}{4} \end{cases} \Rightarrow B = \left(-\infty; -\frac{7}{4}\right)$$

$$D_{f_2} = \left(-\infty; -\frac{7}{4}\right) \cup [2; +\infty)$$




треба да се исклучат од $D_{f_2}$

$$D_f = D_{f_1} \cap D_{f_2} = \left(-\infty; -\frac{7}{4}\right) \cup [2; 4) \cup (4; +\infty)$$

б) $$f(x) = \underbrace{\frac{1}{x^2+3x}}_{f_1(x)} + \underbrace{\ln\left(\frac{3x+1}{2x-5}\right)}_{f_2(x)} \qquad D_f = D_{f_1} \cap D_{f_2}$$

$D_{f_1}$: $x^2+3x \neq 0$

$$x^2+3x = 0 \Rightarrow x(x+3) = 0$$

$$\Rightarrow x_1 = -3, \quad x_2 = 0$$

$$D_{f_1} = \mathbb{R} \setminus \{-3; 0\} = (-\infty; -3) \cup (-3; 0) \cup (0; +\infty)$$

$D_{f_2}$: $\dfrac{3x+1}{2x-5} > 0$ ( $2x-5 \neq 0$ а ова се постигнува со строго неравенство)

$$D_{f_2} = A \cup B$$

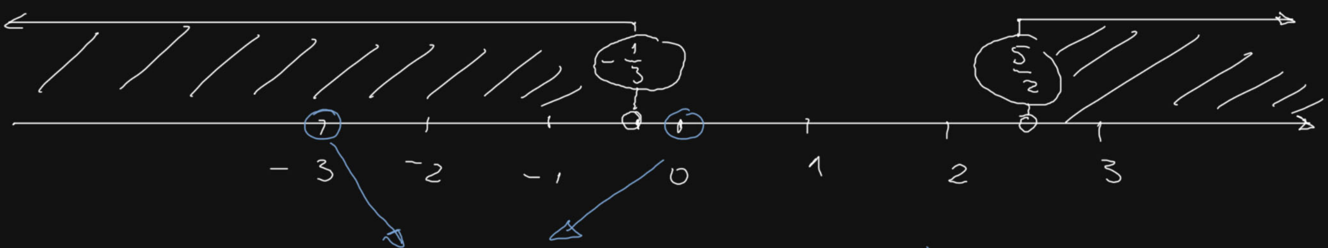
$$A: \begin{cases} 3x+1 > 0 \\ 2x-5 > 0 \end{cases} \Rightarrow \begin{cases} 3x > -1 & /:3 \\ 2x > 5 & /:2 \end{cases} \Rightarrow \begin{cases} x > -\frac{1}{3} \\ x > \frac{5}{2} \end{cases}$$

$$\Rightarrow A = \left(\frac{5}{2}; +\infty\right)$$

$$B: \begin{cases} 3x+1 < 0 \\ 2x-5 < 0 \end{cases} \Rightarrow \begin{cases} 3x < -1 & /:3 \\ 2x < 5 & /:2 \end{cases} \Rightarrow \begin{cases} x < -1/3 \\ x < 5/2 \end{cases}$$

$$\Rightarrow B = \left(-\infty; -\frac{1}{3}\right)$$

$$D_{f_2} = \left(-\infty; -\frac{1}{3}\right) \cup \left(\frac{5}{2}; +\infty\right)$$




треба да се исклучат од $D_{f_2}$

$$D_f = D_{f_1} \cap D_{f_2} = (-\infty; -3) \cup \left(-3; -\frac{1}{3}\right) \cup \left(\frac{5}{2}; +\infty\right)$$

(3) Да се определи инверсна ф-ja на

а) $f(x) = \dfrac{6 + e^{\cos(x/5)}}{1 - e^{\cos(x/5)}}$ (за дома sin наместо cos)

б) $f(x) = \dfrac{\sqrt[7]{e^{8x}} - 2}{\sqrt[7]{e^{8x}} + 4}$

Решение:

$y = f(x)$

$x = f(y) \Rightarrow \qquad y = g(x)$

? дали постои

ако постои, тогаш $g = f^{-1}$

а) $x = \dfrac{6 + e^{\cos(y/5)}}{1 - e^{\cos(y/5)}} \Rightarrow x(1 - e^{\cos(y/5)}) = 6 + e^{\cos(y/5)}$

$\Rightarrow x - x \cdot e^{\cos(y/5)} = 6 + e^{\cos(y/5)}$

$\Rightarrow -x \cdot e^{\cos(y/5)} - e^{\cos(y/5)} = 6 - x$

$\Rightarrow -e^{\cos(y/5)}(x+1) = 6 - x \quad / : -(x+1)$

$\Rightarrow e^{\cos(y/5)} = -\dfrac{6-x}{x+1} = \dfrac{x-6}{x+1} \quad / \ln(\cdot)$

$\Rightarrow \cos\dfrac{y}{5} = \ln\left(\dfrac{x-6}{x+1}\right)$

$\Rightarrow \dfrac{y}{5} = \arccos\left[\ln\left(\dfrac{x-6}{x+1}\right)\right] \quad / \cdot 5$

$\Rightarrow y = 5\arccos\left[\ln\left(\dfrac{x-6}{x+1}\right)\right]$

б) $x = \dfrac{\sqrt[7]{e^{8y}} - 2}{\sqrt[7]{e^{8y}} + 4} \Rightarrow x\left(\sqrt[7]{e^{8y}} + 4\right) = \sqrt[7]{e^{8y}} - 2$

$\Rightarrow x \cdot \sqrt[7]{e^{8y}} + 4x = \sqrt[7]{e^{8y}} - 2$

$\Rightarrow x \cdot \sqrt[7]{e^{8y}} - \sqrt[7]{e^{8y}} = -4x - 2$

$\Rightarrow \sqrt[7]{e^{8y}}\,(x-1) = -4x - 2 \quad / : (x-1)$

$$\Rightarrow \sqrt[7]{e^{8y}} = \frac{-4x-2}{x-1} = -\frac{4x+2}{x-1} = \frac{4x+2}{1-x}$$

$$\Rightarrow e^{\frac{8y}{7}} = \frac{4x+2}{1-x} \quad / \ln(\cdot)$$

$$\Rightarrow \frac{8y}{7} = \ln\left(\frac{4x+2}{1-x}\right) \quad / \cdot \frac{7}{8}$$

$$\Rightarrow y = \frac{7}{8}\ln\left(\frac{4x+2}{1-x}\right)$$

4 Да се пресметаат следните граници

а) $\lim\limits_{x\to+\infty}\left(5x-2-\sqrt{25x^2-9x+8}\right)$ ;

б) $\lim\limits_{x\to+\infty}\left(2x+9-\sqrt{4x^2+13x-5}\right)$ ;

в) $\lim\limits_{x\to 4}\dfrac{x^5-7x^4+12x^3}{tg(x-4)}$ ;  г) $\lim\limits_{x\to 2}\dfrac{tg(x-2)}{x^6-5x^5+6x^4}$

д) $\lim\limits_{x\to+\infty}\left(\dfrac{x-7}{x+3}\right)^{x-2}$ ;  ѓ) $\lim\limits_{x\to+\infty}\left(\dfrac{x+8}{x-5}\right)^{x-3}$

Решение:

а) $$\lim_{x\to+\infty}\left(5x-2-\sqrt{25x^2-9x+8}\right) \overset{(+\infty)-(+\infty)}{=}$$

$$= \lim_{x\to+\infty}\frac{\left(5x-2-\sqrt{25x^2-9x+8}\right)\left(5x-2+\sqrt{25x^2-9x+8}\right)}{5x-2+\sqrt{25x^2-9x+8}}$$

$$= \lim_{x\to+\infty}\frac{(5x-2)^2-\left(\cancel{\sqrt{\phantom{x}}}25x^2-9x+8\right)^{\cancel{2}}}{5x-2+\sqrt{25x^2-9x+8}}$$

$$= \lim_{x\to+\infty} \frac{\cancel{25x^2}-20x+4-\cancel{25x^2}+9x-8}{5x-2+\sqrt{25x^2-9x+8}}$$

$$= \lim_{x\to+\infty} \frac{-11x-4}{5x-2+\sqrt{25x^2-9x+8}} \quad \begin{matrix}(:\frac{1}{x})\\(:\frac{1}{x})\end{matrix}$$

$$= \lim_{x\to+\infty} \frac{-11-\cancel{\frac{4}{x}}^{\to 0}}{5-\cancel{\frac{2}{x}}^{\to 0}+\sqrt{25-\cancel{\frac{9}{x}}^{\to 0}+\cancel{\frac{8}{x^2}}^{\to 0}}}$$

$$= \frac{-11}{5+\sqrt{25}} = \frac{-11}{5+5} = -\frac{11}{10}$$

б) $$\lim_{x\to+\infty}\left(2x+9-\sqrt{4x^2+13x-5}\right) \overset{(+\infty)-(+\infty)}{=}$$

$$= \lim_{x\to+\infty} \frac{(2x+9-\sqrt{4x^2+13x-5})(2x+9+\sqrt{4x^2+13x-5})}{2x+9+\sqrt{4x^2+13x-5}}$$

$$= \lim_{x\to+\infty} \frac{\cancel{4x^2}+36x+81-\cancel{4x^2}-13x+5}{2x+9+\sqrt{4x^2+13x-5}}$$

$$= \lim_{x\to+\infty} \frac{23x+86}{2x+9+\sqrt{4x^2+13x-5}} = \lim_{x\to+\infty} \frac{23+\cancel{\frac{86}{x}}^{\to 0}}{2+\cancel{\frac{9}{x}}^{\to 0}+\sqrt{4+\cancel{\frac{13}{x}}^{\to 0}-\cancel{\frac{5}{x^2}}^{\to 0}}}$$

$$= \frac{23}{2+\sqrt{4}} = \frac{23}{2+2} = \frac{23}{4}$$

в) $$\lim_{x\to 4} \frac{x^5-7x^4+12x^3}{tg(x-4)} = \lim_{x\to 4} \frac{x^3(x^2-7x+12)}{tg(x-4)} \overset{\frac{4^3\cdot 0}{0}=\frac{0}{0}}{=} \quad (*)$$

$$x^2-7x+12=0 \Rightarrow x_{1,2} = \frac{-(-7)\pm\sqrt{(-7)^2-4\cdot1\cdot12}}{2\cdot1} = \frac{7\pm\sqrt{49-48}}{2} = \frac{7\pm1}{2}$$

$$x_1 = \frac{7-1}{2} = 3, \quad x_2 = \frac{7+1}{2} = 4$$

$$(*) = \lim_{x\to 4} \frac{x^3(x-3)(x-4)}{\frac{\sin(x-4)}{\cos(x-4)}} = \lim_{x\to 4} \frac{x^3(x-3)(x-4)\cos(x-4)}{\sin(x-4)}$$

$$= \left[\lim_{x\to 4} \frac{x-4}{\sin(x-4)}\right] \cdot \underbrace{\lim_{x\to 4} x^3(x-3)\cos(x-4)}$$

$$= 4^3\cdot(4-3)\underbrace{\cos(4-4)}_{=\cos 0 = 1} = 4^3\cdot 1\cdot 1 = 64$$

$$= \left| \begin{array}{l} \text{смена за I лимес} \\ x-4 = t \\ x\to 4,\ t\to 0 \end{array} \right| = 64\cdot\lim_{t\to 0}\frac{t}{\sin t} = 64\cdot 1 = 64$$

ѓ) – за дома (одговор $-\frac{1}{16}$)

д) $$\lim_{x\to+\infty}\left(\frac{x-7}{x+3}\right)^{x-2} = \lim_{x\to+\infty}\left(1+\frac{x-7}{x+3}-1\right)^{x-2}$$

$$= \lim_{x\to+\infty}\left(1+\frac{x-7-(x+3)}{x+3}\right)^{x-2} = \lim_{x\to+\infty}\left(1+\frac{\cancel{x}-7-\cancel{x}-3}{x+3}\right)^{x-2}$$

$$= \lim_{x\to+\infty}\left(1+\frac{-10}{x+3}\right)^{x+3-5} = \left[\lim_{x\to+\infty}\left(1+\frac{-10}{x+3}\right)^{x+3}\right]\cdot\underbrace{\left[\lim_{x\to\infty}\left(1+\frac{-10}{x+3}\right)\right]^{-5}}_{=1^{-5}=1}$$

$$= \left| \begin{array}{l} \text{смена за I лимес:} \\ x+3 = t \\ x\to+\infty \Rightarrow t\to+\infty \end{array} \right| = \left[\lim_{t\to+\infty}\left(1+\frac{-10}{t}\right)^t\right]\cdot 1$$

$$= e^{-10}\cdot 1 = \frac{1}{e^{10}}$$

е) за дома (одговор: $e^{13}$)